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Airy GB (1861). On the algebraical and numerical theory of errors of observations and the combination of observations. Cambridge: MacMillan and co.

Title pages

ON THE

ALGEBRAICAL AND NUMERICAL

THEORY

OF

ERRORS OF OBSERVATIONS

AND THE

COMBINATION OF OBSERVATIONS.

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MACMILLAN AND CO.

Cambridge:

AND 23, HENRIETTA STREET, COVENT GARDEN,

London.

1861.

Key passages

PREFACE.

THE Theory of Probabilities is naturally and strongly divided into two parts. One of these relates to those chances which can be altered only by the changes of entire units or integral multiples of units in the fundamental conditions of the problem; as in the instances of the number of dots exhibited by the upper surface of a die, or the numbers of black and white balls to be extracted from a bag. The other relates to those chances which have respect to insensible gradations in the value of the element measured; as in the duration of life, or in the amount of error incident to an astronomical observation.

It may be difficult to commence the investigations proper for the second division of the theory without referring to principles derived from the first. Nevertheless, it is certain that, when the elements of the second division of the theory are established, all reference to the first division is laid aside; and the original connexion is, by the great majority of persons who use the second division, entirely forgotten. The two divisions branch off into totally unconnected subjects; those persons who habitually use one part never have occasion for the other; and practically they become two different sciences.

In order to spare astronomers and observers in natural philosophy the confusion and loss of time which are produced by referring to the ordinary treatises embracing both branches of Probabilities, I have thought it desirable to draw up this tract, relating only to Errors of Observation, and to the rules, derivable from the consideration of these Errors, for the Combination of the Results of Observations. I have thus also the advantage of entering somewhat more fully into several points, of interest to the observer, than can possibly be done in a General Theory of Probabilities.

No novelty, I believe, of fundamental character, will be found in these pages. At the same time I may state that the work has been written without reference to or distinct recollection of any other treatise (excepting only Laplace's *Théorie des Probabilités*); and, the methods of treating the different problems may therefore differ in some small degrees from those commonly employed.

G. B. AIRY.

ROYAL OBSERVATORY, GREENWICH,
January 22, 1861.

PRINCIPLES OF FORMING THE MOST ADVANTAGEOUS COMBINATION OF FALLIBLE MEASURES.

§ 10. *Method of combining measures; meaning of "combination-weight;" principle of most advantageous combination: caution in its application to "entangled measures."*

62. THE determinations of physical elements from numerous observations, to which this treatise relates, are of two kinds.

The First is, the determination of some one physical element, which does not vary or which varies only by a certainly calculable quantity during the period of observations, by means of numerous *direct* and *immediate* measures. Thus, in the measure of the apparent angular distance between the components of a double star, we are making direct and immediate measures of an invariable quantity; in measuring the difference of Moon's right ascension from the right ascension of known stars at two or more known stations, in order to render similar observations at an unknown station available for determining its longitude, we are making direct and immediate measures of quantities which are different at the two or more stations, but whose difference can be accurately computed.

63. The measures thus obtained are all fallible, and the problem before us is, How they shall be combined? It is not inconceivable that different rules might be adopted for this purpose, depending (for instance) upon the products of different powers of the various measures, and ultimate extraction of the root corresponding to the sum of the indices of powers: or upon other imaginable operations. But the one method (to which all others will approximate in effect) which has universally recommended itself, not only by its simplicity, but also by the circumstance that it permits all the measures to be increased or diminished by the same quantity (which is sometimes convenient) is, to multiply each measure by a number (either different for each different measure, or the same for any or all) which number is here called the "combination-weight;" to add together these products of measures by combination-weights; and to divide the sum by the sum of combination-weights.

64. The problem of advantageous combination now becomes this, What combination-weights will be most advantageous? And to answer this, we must decide on the criterion of advantage. The criterion on which we shall fix is:—That combination is best which gives a result whose *probable error*, or mean error, or error of mean square, is the smallest possible. This is all that we can do. We cannot assert that our result shall be correct; or that, in the case before us, its *actual error* shall be small, or smaller than might be given by many other combinations; but we can assert that it is *probable* that its actual error will be the smallest, and that it is *certain* that, by adopting this rule in a very great number of instances, we shall on the whole obtain results which are liable to smaller errors than can be obtained in any other way.

65. Now if we know the probable errors, or the proportion of probable errors, of the individual observations, (an indispensable condition,) we can put known symbols for them, and we can put undetermined symbols for the combination-weights; and, by the precepts of Part II, we can form the symbolical expression for the probable error of the result. This probable error is to be made minimum, the undetermined quantities being the combination-weights. Thus we fall upon the theory of *complex maxima and minima*. Its application is in every case very easy, because the quantities required enter only to the second order. Instances will be found in Articles 68 to 72.

66. It sometimes happens that, even in the measures of an invariable quantity, combinations of a complicated character occur. Different complex measures are sometimes formed, leading to the same result; in which some of the observations are different in each measure, but other observations are used in all or in several of the measures; and thus the measures are not strictly independent. We shall call these "entangled measures." The only caution to be impressed on the reader is, to be very careful, in forming the separate results, to delay the exhibition of their probable errors to the last possible stage; expressing first the *actual* error of each separate result of the form ultimately required, by the *actual* error of each observation. It will often be found that, in this way, the results of observations will be totally or partially eliminated (and justly so), which, if the probable errors had been formed at an early stage, would have vitiated the result. Instances of this will be given below (Articles 74 to 85).

67. The Second class is, the simultaneous determination of several physical elements. It may be illustrated by one of its most frequent applications, that of determining the corrections to be applied to the orbital elements of a planet's orbit. The quantities measured are right ascensions and north polar distances, observed when the planet is at different points in its orbit, and in different positions with respect to the observer. If approximate orbital elements are adopted, each having an indeterminate symbol attached to it for the small correction which it may require; it will be possible to express, by orbital calculation, every right ascension and north polar distance by numerical quantities, to which are attached definite multiples of the several indeterminate symbols. Equating these to the observed right ascensions and north polar distances, a long series of numerous equations is obtained, with different multiples of the indeterminate symbols; each equation being subject to its own *actual* error of observation. And the question before us is now, How shall these numerous equations be combined so as to form exactly as many equations as the number of indeterminate symbols, securing at the same time the condition that the probable error of every one of the values thus obtained shall be the smallest possible? This is also a case of *complex maxima and minima*. Numerous problems in astronomy, geodesy, and other applied sciences, require this treatment. It will be fully explained in Articles 87 to 105.